ALLOCATION OF EFFORT

IN A STRATIFIED SURVEY DESIGN

Douglas C. Heard

Department of Renewable Resources

Government of the N.W.T.

Yellowknife, N.W.T.

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ABSTRACT

To determine the number of transects one should cover within each stratum in order to get the most precise population estimate possible for a given amount of flying, the procedure is:

1) Determine the total miles available to fly on transect (M), the average transect length (W) in each stratum and the approximate number of animals in each stratum (Y);

2) Substitute those data in the following equation and solve for n_h, the number of transects in stratum h,

$$n_h = \frac{(Y_h)(M)}{(W_h)(\sqrt{W_h})[\Sigma(Y_h/\sqrt{W_h})]} \qquad \text{(equation 7.3 from text)}$$

3) Check results to see if the sum of number of transects times their length equals the miles available to do the survey.

$$\Sigma(n_h)(W_h) = M \qquad \text{(equation 7.1 from text)}$$

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INTRODUCTION

This paper describes how to determine the number of transects one should cover within each stratum in order to get the most precise population estimate possible for a given amount of flying. Norton-Griffiths (1975) described how to use aerial strip transects to estimate animal abundance and I assumed that readers are familiar with the technique and with Norton-Griffith's terminology. I wrote this note because I was confused by Norton-Griffith's description of how to allocate survey effort among strata and I thought a more detailed description and some worked examples would help other researchers.

RESULTS & DISCUSSION

Determining the Samples Size in Each Stratum

The number of transects surveyed per stratum (n_h) should be proportional to:

$$(N_h)\ (S_h)\ /\sqrt{C_h} \qquad \text{(equation 1.1)}$$

(Snedecor and Cochran 1967:523, Cochran 1977:97) where N_h is the number of transect lines in stratum h that would give 100% coverage, S_h is the standard deviation of the mean density among transects in the h^{th} stratum, and C_h is the cost per sampling unit in the h^{th} stratum. This method of allocation gives the smallest standard error of the population estimate for a given cost. The equation indicates that more samples should be taken in large (N_h large) strata that have clumped (variable) caribou densities (S_h large), and fewer samples where sampling is unusually expensive (C_h large). The relationship is consistent with common sense.

The cost per sampling unit excluding positioning requirements is directly proportional to the average stratum width (W) because transects should always be oriented perpendicular to the long axis of the stratum (Norton-Griffiths 1975). Equation 1.1 can be rewritten as:

$$n_h \propto (N_h)\ (S_h)\ /\sqrt{W_h} \qquad \text{(equation 1.2)}$$

In order to apply this formula, it is necessary to know (1) the strata boundaries, and (2) the standard deviation of the mean density in each stratum. Stratum boundaries can be determined from a preliminary "systematic transect" survey. Because calculation of the stratum standard deviation can be assumed to be positively and linearly correlated with animal density (Caughley 1977:28, Norton-Griffiths 1975:36), it is expedient (because S_h is difficult to calculate) to insert the estimate of animal density (R_h) in place of S_h in equation 1.2, producing:

$$n_h \propto (N_h)\ (R_h)\ /\sqrt{W_h} \qquad \text{(equation 1.3)}$$

or $n_h = (k)\ (N_h)\ (R_h)\ /\sqrt{W_h}$, where k is a constant.

The stratum area (Z_h) is equal to the mean width (W_h) times the length (l_h). Thus:

$$Z_h = (l_h)(W_h) \qquad \text{(equation 2)}$$

Stratum density (R_h) is equal to the stratum population estimate (Y_h) divided by the stratum area (Z_h). Thus:

$$R_h = Y_h\ /\ Z_h \qquad \text{(equation 3.1)}$$

Substituting equation 2 into equation 3.1 yields:

$$R_h = Y_h\ /\ (l_h)\ (W_h) \qquad \text{(equation 3.2)}$$

If transect width is 1/2 mile, 1/4 mile on each side of the aircraft, and transects are oriented perpendicular to the long axis of the stratum, then N_h = 2 x maximum length of stratum (l_h) in miles. The general form of this equation is

$$N_h = (c)\ (l_h) \qquad \text{(equation 4)}$$

where c is a constant.

Substituting for N_h (equation 4) and R_h (equation 3.2) in equation 1.3 yields:

$$n_h = (k)\ (c)\ (l_h)\ (Y_h)\ /\ (l_h)\ (W_h)\ (\sqrt{W_h}) \qquad \text{(equation 1.4a)}$$

Which simplifies to:

$n_h = (k)\ (Y_h)\ /\ (W_h)\ (\sqrt{W_h})$ (equation 1.4)

because k and c can still be represented by k.

If we let

$p_h = Y_h\ /\ (W_h)\ (\sqrt{W_h})$ (equation 5)

for now and substitute p_h to equation 1.4

then $n_h = (k)\ (p_h)$ (equation 1.5)

The ratio between the number of transects in any two strata (h and x) is:

$$\frac{n_h}{n_x} = \frac{(k)\ (p_h)}{(k)\ (p_x)} = \frac{p_h}{p_x} \text{ or } n_h = (n_x)(p_h)\ /\ (p_x) \quad \text{(equation 6)}$$

We now know how to calculate the relative sample sizes in each stratum. To determine the actual sample sizes we incorporate the resources available to do the census. The limiting resource is almost always aircraft flying time, or more specifically, the number of transect miles one can cover. If one has 30 hours of aircraft hours available the transect miles available are computed by multiplying the speed of the aircraft times the hours available and subtracting positioning requirements and associated non-transect flying. The miles flown in each stratum will be the number of transects (n_h) times the average length (W_h). The sum over all strata must equal M, thus:

$$[(n_1)\ (W_1)] + [(n_2)\ (W_2)] + [(n_3)\ (W_3)] + \ldots = M$$

or

$\sum(n_h)\ (W_h) = M$ (equation 7.1)

If we present all transect ratios after equation 6 as functions of n_1 then:

$$n_2 = n_1p_2/p_1$$

$$n_3 = n_1p_3/p_1$$

$$n_4 = n_1p_4/p_1$$

and substitute these relationships into equation 7.1 we can solve for n_1 such that:

$$[(n_1\ (w_1)] + [(n_1)\ (p_2)\ (\ w_2)\ /\ p_1] +$$
$$[(n_1)\ (p_3)\ (w_3)\ /\ p_1] + \ldots = M$$

and

$$n_1 = \frac{M}{(w_1) + (w_2p_2\ /\ p_1) + w_3p_3\ /\ p_1 + \ldots)}$$

$$= (p_1M)\ /\ (w_1p_1 + w_2p_2 + w_3p_3 + \ldots)$$

$$= (p_1M)\ /\ \sum(W_hp_h) \qquad \text{(equation 7.2)}$$

Substituting for p_h and p_1 from equation 5 yields:

$$n_1 = (Y_1)\ (M)\ /\ (W_1)\ (\sqrt{W_1})\ [\sum(Y_h\ /\sqrt{W_h})] \text{(equation 7.3)}$$

Thus, n is related to the miles available, the average transect length, and the estimated number of animals in the strata.

Once a value for n_1 is calculated, n_2 and n_3 are obtained in the same way. Note that $M/\sum(Y_h\ /\sqrt{W_h})$ appears in all calculations.

If the actual estimate of the stratum standard deviation (S_h) is available, it is preferable to use this value directly rather than replace it with an estimate based on density. In this situation, the calculations of n_1 would be as follows:

$$n_1 = \frac{(S_1)\ (l_1)\ (M)}{(\sqrt{W_1})\ [\sum(W_h)\ (S_h)\ (l_h)\ /(\sqrt{W_h})]} \qquad \text{(equation 8)}$$

A further rule of thumb is that the number of sample units should never be less than 5 (Gasaway et al. 1981), but Norton-Griffiths (1975:35) suggests that the number of sample units should be between 10 and 50. I find it impractical to attempt greater than 50% coverage.

Examples of Sample Size Determination Calculations

Example 1.

If there are two strata, and 100 miles available to fly the transects (i.e. excluding positioning requirements) and

$y_1 = 22$, $W_1 = 5$ miles, $l_1 = 20$ miles, $S_1 = 0.22$, and

$y_2 = 11$, $W_2 = 2$ miles, $l_2 = 50$ miles, $S_2 = 0.11$.

Substituting into equation 7.3 yields:

$$n_1 = (22)\ (100)\ /\ (5)\ (\sqrt{5})\ [22/\sqrt{5}) + (11/\sqrt{2})]$$

$$= [(22)\ /\ (5)\ (\sqrt{5})]\ [5.68]$$

$$= 11$$

and

$$n_2 = (11)\ (100)\ /\ (2)\ (\sqrt{2})\ [(22\ /\sqrt{5}) + (11\ /\sqrt{2})]$$

$$= [(11)\ /\ (2)\ (\sqrt{2})]\ [5.68]$$

$$= 22$$

Thus, 11 transects should be flown in stratum 1 and 22 in stratum 2. A test of the results is obtained by substituting back into equation 7.1. Doing this we find that 22 x 2 = 44 miles will be flown in stratum 2 and 11 x 5 = 55 miles will be flown in stratum 1. Transect flights total 99 miles, which is equal to the resources available.

If equation 8 is used

$$n_1 = \frac{(0.22)\ (20)\ (100)}{(\sqrt{5})\ [((5)\ (0.22)\ (20)/\sqrt{5})\ +((2)\ (0.11)\ (50)/\sqrt{2})]}$$

$= 440 / 39.4$

$= 11.2$ as before

Example 2.

If M = 900 miles

$Y_1 = 28000$ $Y_2 = 22000$ $Y_3 = 3000$

$W_1 = 17$ miles $W_2 = 9$ miles $W_3 = 15$ miles

Then $n_1 = \dfrac{(28000)\ (900)}{(17)\ (\sqrt{17})\ \left(\dfrac{28000}{\sqrt{17}} + \dfrac{22000}{\sqrt{9}} + \dfrac{3000}{\sqrt{15}}\right)}$ (equation 7.3)

$$= \frac{(28000)}{(17)\ (\sqrt{17})\ (16.6)}$$

$= 20.3$

$$n_2 = \frac{(22000)}{(9)\ (\sqrt{9})\ (16.6)}$$

$= 49$

$$n_3 = \frac{(3000)}{(15)\ (\sqrt{15})\ (16.6)}$$

$= 3$

But if we take Norton-Griffith's advice and fly at least 10 transects in stratum 3 ($n_3 = 10$), we must reduce n_1 and n_2 to provide the required resources. If $n_3 = 10$ then remaining flying is:

900 - (10) (15) = 750 miles and

$$n_1 = \frac{(28000)\ (750)}{(17)\ (\sqrt{17})\ \left(\frac{28000}{\sqrt{17}} + \frac{22000}{\sqrt{19}}\right)}$$

$$= 21$$

and $n_2 = 43$

Check: $n_1M_1 + n_2M_2 + n_3M_3 = M$

$$\text{L.S.} = (21)\ (17) + (43)\ (9) + (10)\ (15)$$

$$= 894$$

$$\doteq M$$

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